Thermal Two Point Function of a Heavy Muon in hot QED plasma within Bloch Nordsieck Approximation

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Abstract

The thermal propagator of a heavy muon propagating in a hot QED plasma is examined within the Bloch-Nordsieck approximation, which is valid in the infrared region. It is shown that the muon damping rate is finite, in contrast to the lower-order calculation with hard thermal loop resummations taken into account.

11.10.Wx, 12.20.-m, 12.38.Lg, 13.35.Bv

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I. INTRODUCTION

Because of much interest in the early Universe as well as in the quark-gluon plasma to be produced in heavy-ion collisions, many authors have studied gauge theories at high temperature. Thermal propagators of bosons in real-time thermal field theory carry the Boltzmann factor $1/(e^{E/T}-1)$. For a massless boson like gauge boson, this factor behaves as $\sim T/k$ at the infrared region, $k \ll T$, which leads to stronger infrared divergences in thermal amplitudes than in vacuum theory. In vacuum theory, it is well known [1–3] that all the physical quantities are free from infrared singularities, provided one sums over all degenerate final states (Bloch Nordsieck mechanism). An important exception to this general statement is the so-called Coulomb singularity. For thermal reaction rates, due to the stronger infrared singularities mentioned above, it is not known if they do not possess infrared singularities, except again for Coulomb singularities. Up until now, a proof is available of absence of leading infrared singularities in a generic thermal reaction rate [4] and of all infrared singularities in a bremsstrahlung process within some specific approximation [5].

It has been established [6,7,14] that, to obtain a consistent perturbation expansion, resummations of hard-thermal-loop (HTL) contributions are necessary. According to this HTL resummation scheme, infrared behaviors of thermal amplitudes are softened or screened compared with those in naive perturbative calculation. this softening renders some otherwise divergent physical quantities finite [7,8]. There are, however, some other physical quantities which are still infrared divergent to leading order in HTL resummation scheme. A well-known example is the quark (muon) damping rate in a hot QCD (QED) plasma [9]. As a matter of fact, the quark (muon) damping rate diverges due to the Coulomb singularity. In naive perturbation calculation, the divergence is of power type. In HTL resummation scheme, as mentioned above, the infrared behaviour is softened and the infrared divergence becomes to be of logarithmic type. It is expected, at least for quark case, that the infrared singularity is screened when still higher order resummation is performed. As far as the infrared behaviour of a spinor field in QED is concerned, Bloch-Nordsieck approximation is a good approximation [10,11]. In this paper, the thermal two-point function of a heavy muon in a QED plasma is computed within this approximation. Various properties of the result are discussed, among those is an especially important conclusion: The decaying behavior of a heavy muon does not suffer from divergence.

II. THERMAL TWO POINT FUNCTION OF A HEAVY MUON

By the term "heavy muon" we mean so heavy a muon that it is not thermalized, i.e., $e^{-E/T} \ll 1$. The hot QED plasma means that the temperature T of the plasma is much higher than the electron mass m, $T \gg m$. As discussed in Sec. I, the infrared behavior of the theory is responsible for the divergent damping rate. Thus, we employ the Bloch-Nordsieck approximation to the muon sector of the QED Lagrangian. The Lagrangian density is obtained [10,11] by substituting a constant vector u^μ ($u^2=1$) for γ^μ in the muon sector of the standard QED Lagrangian density:

$$\mathcal{L}^{total} = \mathcal{L}_\Psi + \mathcal{L}_\psi + \mathcal{L}_A \, , \tag{2.1}$$

$$\mathcal{L}_\Psi = \overline{\Psi}(iu \cdot \partial + eu \cdot A - M)\Psi \ , \tag{2.2}$$

$$\mathcal{L}_\psi = \overline{\psi}(i\gamma \cdot \partial + e\gamma \cdot A - m)\psi \ , \tag{2.3}$$

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \ , \tag{2.4}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ , \tag{2.5}$$

where Ψ, ψ, M, m, and A stand, respectively, for heavy muon field, electron field , muon mass, electron mass and photon field. It is known that, in the infrared region, the spinor structure of the heavy muon does not play any significant roles [10,11]. Then the above Lagrangian density well approximates the exact QED Lagrangian in the infrared region.

To obtain the thermal two-point function of the heavy muon, we start from the Euclidean or imaginary-time formalism of thermal field theory, initiated by Matsubara [12], in which a time arrow flows from zero to $-i/T$ in a complex time plane. In this formalism, the energy is discrete; $p_4 = 2\pi nT$ $(n = 0, \pm 1, \pm 2, \cdots)$ for a boson field and $p_4 = \pi(2n+1)T$ for a fermion field. At the final stage, we analytically continue the two-point function, thus obtained, to real time (or real energy). A four vector $a^\mu = (\vec{a}, a_4)$ in the imaginary-time formalism is obtained from the counter part $(a_0, \vec{a})$ in Minkowski space by an analytic continuation, $a_0 \longrightarrow -ia_4$. Similar continuation is made for tensors.

The generating functional is introduced as usual,

$$Z = \int \mathcal{D}\overline{\Psi}\mathcal{D}\Psi\mathcal{D}\overline{\psi}\mathcal{D}\psi\mathcal{D}A \exp S \ , \tag{2.6}$$

$$\begin{aligned} S = \int_0^\beta d^4x \Big(& \mathcal{L}^{total} - \frac{1}{2\lambda}(\partial \cdot A)^2 \\ & + \overline{\eta}\Psi + \overline{\Psi}\eta + \overline{\zeta}\psi + \overline{\psi}\zeta + j \cdot A \Big) \quad , \end{aligned} \tag{2.7}$$

where the covariant gauge is employed. In Eq. (2.7), $\eta, \zeta, j, \cdots$ are sources being conjugate to the fields with which they couple. After integrating over $\overline{\Psi}, \Psi, \overline{\psi}$, and ψ in Eq. (2.6), the two point function of the heavy muon is obtained as,

$$\begin{aligned} G_E &= -\frac{1}{Z}\frac{\delta}{\delta\overline{\eta}(x)}\frac{\delta}{\delta\eta(y)} \ Z[\eta, \overline{\eta}, \zeta, \overline{\zeta}, j]\Big|_{\eta,\cdots=0} \\ &= \frac{1}{Z}\int \mathcal{D}A \det(iu_E \cdot \partial - eu_E \cdot A - M) \\ &\quad \times \det(i\gamma \cdot \partial - e\gamma \cdot A - m) G(x-y; eA, M) \\ &\quad \times \exp\left[-\frac{1}{4}\int d^4x F_{\mu\nu}F_{\mu\nu} - \frac{1}{2\lambda}(\partial \cdot A)^2\right] \ , \end{aligned} \tag{2.8}$$

where

$$(iu_E \cdot \partial - eu_E \cdot A - M)G(x-y; eA, M) \equiv \delta_E(x-y) \, , \tag{2.9}$$

$$\delta_E(x-y) \equiv T\sum_{k_4=odd}\int \frac{d^3k}{(2\pi)^3}\exp[ik \cdot (x-y)] \quad , \tag{2.10}$$

$$k_4 = \pi(2n+1)T \qquad (n = 0, \pm 1, \pm 2, \cdots) \quad . \tag{2.11}$$

In Eq. (2.10), "$k_4 = odd$" indicates that the summation runs over n as defined in Eq. (2.11).

Since the heavy muon is not thermalized, the muon determinant has been reduced to $\det(iu_E \cdot \partial - M)$ as in vacuum theory [10]. Using the formula $\det a = \exp(\mathrm{Tr} \ln a)$ and Furry's theorem [13], we have

$$\det(i\gamma \cdot \partial - e\gamma \cdot A - m) = \det(i\gamma \cdot \partial - m) \exp\Big[-\frac{e^2}{2}\mathrm{Tr} \int d^4z_1 d^4z_2 (\gamma \cdot A) G_0(z_1 - z_2; m) \times (\gamma \cdot A) G_0(z_2 - z_1; m) + O(e^4)\Big] , \tag{2.12}$$

$$G_0(x - y; m) = -T \sum_{k_4 = even} \int \frac{d^3k}{(2\pi)^3} \frac{\exp[ik \cdot (x - y)]}{\gamma \cdot k + m} \quad . \tag{2.13}$$

Here $G_0(x - y; m)$ denotes a bare electron propagator. As seen from Eq. (2.8) with Eq. (2.12), in the heavy-muon two-point function, the contributions of electrons enter through thermal electron loops. Here we employ the HTL resummation scheme for the electron sector: To leading order, among such electron loops, only HTLs that yield corrections to soft-photon external lines are important. In hot QED considered here, the $O(e^4)$ term in Eq. (2.12) vanishes [14] to leading order in the HTL resummation scheme. We then ignore it in the following.

Following [10], we introduce the integral representation,

$$G(x - y; eA, M) = -i \int_0^\infty d\tau \exp\Big[i\tau(u_E \cdot \partial - eu_E \cdot A - M + i\epsilon_c)\Big] \delta_E(x - y) , \tag{2.14}$$

where ϵ_c is a "convergence" factor. Defining the function $U(\tau)$ as,

$$U(\tau) \equiv \exp\Big[i\tau(iu_E \cdot \partial - eu_E \cdot A - M + i\epsilon_c)\Big] \delta_E(x - y) , \tag{2.15}$$

$$G(x - y; eA, M) \equiv -i \int_0^\infty d\tau \, U(\tau) , \tag{2.16}$$

we have

$$-i\frac{\partial U}{\partial \tau} = (iu_E \cdot \partial - eu_E \cdot A - M + i\epsilon_c) U(\tau) \quad , \tag{2.17}$$

$$U(0) = \delta_E(x - y) \quad . \tag{2.18}$$

Now we assume the following form for the solution to Eq. (2.17) with Eq. (2.18),

$$U(\tau) = T \sum_{q_4 = odd} \int \frac{d^3q}{(2\pi)^3} \exp\Big[iK(x - y|A, \tau) + iq \cdot (x - y) - i\tau(M + u_E \cdot q - i\epsilon_c)\Big] \quad . \tag{2.19}$$

Eq. (2.17) with Eq. (2.19) leads to

$$-\frac{\partial K}{\partial \tau} = u_E \cdot \frac{\partial K}{\partial x} + e u_E \cdot A \ , \tag{2.20}$$

$$K(\tau = 0) = 0 \ , \tag{2.21}$$

where $u_E = (\vec{u}, u_4)$. It is straightforward to solve this equation in the form:

$$K(x-y|A,\tau) = -eT \sum_{k_4=even} \int \frac{d^3k}{(2\pi)^3} \exp[ik\cdot(x-y)]A(k)\cdot u_E \int_0^\tau d\tau' \exp[-iu_E\cdot k\tau'] \ , \tag{2.22}$$

$$k_4 = 2\pi n T \quad (n = 0, \pm 1, \pm 2, \cdots) \ , \tag{2.23}$$

where "$k_4 = even$" indicates that the summation runs over n as defined in Eq. (2.23).

Using all the formulae displayed above, and carrying out the integration by part, we obtain for G_E in Eq. (2.8):

$$\begin{aligned} G_E = i \int \mathcal{D}A \int_0^\infty d\tau T \sum_{q_4=odd} \int \frac{d^3q}{(2\pi)^3} \exp\Bigg[& iq\cdot(x-y) - i\tau(M + u_E\cdot q - i\epsilon_c) \\ & -\frac{e^2}{2}\int d^4z_1 d^4z_2 \mathrm{Tr}\,(\gamma\cdot A)G(z_1-z_2; e=0, m)(\gamma\cdot A)G(z_2-z_1; e=0, m) \\ & + \int d^4z \frac{1}{2}A^\mu \left\{\delta^{\mu\nu}\partial^2 - \left(1-\frac{1}{\lambda}\right)\partial^\mu\cdot\partial^\nu\right\} A^\nu \\ & -ieT \sum_{k_4=even}\int \frac{d^3k}{(2\pi)^3} A(k)\cdot u_E \exp[ik\cdot(x-y)] \int_0^\tau d\tau' \exp[-iu_E\cdot k\tau']\Bigg] \ . \end{aligned} \tag{2.24}$$

Going to the momentum space,

$$\mathcal{G}_E = \int d^4(x-y) \exp[-ip\cdot(x-y)] G_E \ , \tag{2.25}$$

we have,

$$\begin{aligned} \mathcal{G}_E = i \int \mathcal{D}A \int_0^\infty d\tau \exp\Bigg[& -i\tau(M - u_E\cdot p - i\epsilon_c) \\ & -\frac{T}{2}\sum_{l_4=even}\int \frac{d^3l}{(2\pi)^3}\, A^\mu(l)\left\{\delta^{\mu\nu}l^2 - \left(1-\frac{1}{\lambda}\right)l^\mu l^\nu + \Pi^{\mu\nu}(l)\right\} A^\nu(-l) \\ & -ieT\sum_{l_4=even}\int \frac{d^3l}{(2\pi)^3}\exp[il\cdot u_E\tau]A(l)\cdot u_E \int_0^\tau d\tau' \exp[-il\cdot u_E\tau']\Bigg] \ , \end{aligned} \tag{2.26}$$

where $\Pi^{\mu\nu}$ is the thermal vacuum polarization tensor of the photon:

$$\begin{aligned} \Pi^{\mu\nu}(l) &= e^2 T \sum_{k_4=odd}\int \frac{d^3k}{(2\pi)^3}\mathrm{Tr}\gamma^\mu \frac{1}{\gamma\cdot(k-l)+M}\gamma^\nu \frac{1}{\gamma\cdot k + M} \\ &= \left(\hat{\delta}^{\mu\nu} - \frac{\kappa^\mu\kappa^\nu}{\kappa^2}\right)\Pi_T + \left(\delta^{\mu 0}\delta^{\nu 0} + \frac{\kappa^\mu\kappa^\nu}{\kappa^2} - \frac{l^\mu l^\nu}{l^2}\right)\Pi_L \ , \end{aligned} \tag{2.27}$$

where $\hat{\delta}^{\mu\nu} = \mathrm{diag}\,(1,1,1,0)$ and $\kappa^\mu = (\vec{l}, 0\,)$. In Eq. (2.27), Π_T (Π_L) is the transverse (longitudinal) component of $\Pi^{\mu\nu}$ (See Appendix A).

The exponent in Eq. (2.26) is quadratic with respect to A, and we can perform the functional integration over A:

$$\mathcal{G}_E = i\int_0^\infty d\tau \exp\left[-i\tau(M + u_E\cdot p - i\epsilon_c) + \frac{\alpha}{2\pi^2}D(\tau, u_E)\right] , \tag{2.28}$$

$$\begin{aligned} D(\tau, u_E) = &-(2\pi\mu)^{3-D}T\sum_{l_4=even}\int d^D l\;\frac{1}{(l\cdot u_E)^2}\Big\{1-\cos(l\cdot u_E\tau)\Big\} \\ &\times\left[\frac{1}{l^2+\Pi_T}\left\{\vec{u}^2 - \frac{(\vec{l}\cdot\vec{u})^2}{\vec{l}^2}\right\} + \frac{1}{l^2+\Pi_L}\left\{{u_E}^2 - \frac{(l\cdot u_E)^2}{l^2} - \vec{u}^2 + \frac{\left(\vec{l}\cdot\vec{u}\right)^2}{\vec{l}^2}\right\}\right. \\ &\left.+\lambda\left(\frac{l\cdot u_E}{l^2}\right)^2\right] , \end{aligned} \tag{2.29}$$

where α is the fine structure constant. Here, in order to regulate the ultraviolet divergence, we have employed the dimensional regularization, $D = 3-\epsilon$ with $\epsilon > 0$. (A dimensionful parameter μ is introduced, as usual, so that e remains dimensionless.) In the HTL approximation, which we employ for the electron sector, the explicit form of Π_T and Π_L are known [6–9,15]:

$$\Pi_T = -\frac{1}{2}{m_{sc}}^2\frac{{l_4}^2}{\vec{l}^2}\left\{1 - \left(1+\frac{\vec{l}^2}{{l_4}^2}\right)\frac{il_4}{2|\vec{l}|}L(il_4, |\vec{l}|)\right\} , \tag{2.30}$$

$$\Pi_L = {m_{sc}}^2\frac{l^2}{\vec{l}^2}\left\{1 - \frac{il_4}{2|\vec{l}|}L(il_4, |\vec{l}|)\right\} , \tag{2.31}$$

where

$$m_{sc} = \frac{eT}{\sqrt{3}} , \tag{2.32}$$

$$L(il_4, |\vec{l}|) = \ln\frac{il_4 + |\vec{l}|}{il_4 - |\vec{l}|} . \tag{2.33}$$

m_{sc} in Eq. (2.32) is thermal or Debye mass of the thermal photon. In obtaining Eqs. (2.29) – (2.33), the electron mass m has been ignored (cf. the first paragraph).

Let us turn back to $D(\tau, u_E)$ in Eq. (2.29). After a straightforward but tedious calculation (cf. Appendix A), we have

$$\begin{aligned} D(\tau, u_E) = &-2\pi^2 T\tau\frac{{u_4}^2}{|\vec{u}|}\left(\frac{1}{\epsilon} + \ln\frac{2\pi\mu}{\sqrt{\pi}m_{sc}} - \frac{\gamma}{2} - \frac{1-e^{-m_{sc}|\vec{u}|\tau}}{|\vec{u}|\tau m_{sc}} + E_i(-|\vec{u}|\tau m_{sc})\right) \\ &-2\pi^2 T\tau|\vec{u}|\left(\frac{1}{\epsilon} + \frac{\gamma}{2} - 1 + \ln\frac{2\pi\mu|\vec{u}|\tau}{\sqrt{\pi}}\right) - (\lambda-1)\pi^2 T\tau|\vec{u}| \\ &+2\pi^2 T\tau\frac{{u_E}^2}{|\vec{u}|}\left(\frac{1}{\epsilon} - \frac{\gamma}{2} - \ln\sqrt{\pi}\right) + \frac{2\pi}{\epsilon} + \pi\gamma - 2\pi\ln\frac{4\pi^{3/2}T}{2\pi\mu} - \frac{2\pi u_4}{|\vec{u}|}\tan^{-1}\frac{u_4}{|\vec{u}|} \end{aligned}$$

$$
\begin{aligned}
&-2\pi^2 T\tau\frac{{u_E}^2}{|\vec{u}|}\left(\ln\frac{T}{2\pi\mu}+\ln\frac{u_E}{|\vec{u}|}\right)+4\pi^2 T\tau\frac{{u_E}^2}{|\vec{u}|}\int_1^\infty dx\frac{1}{x}\frac{1}{e^{2\pi T|\vec{u}|\tau x}-1}\\
&+\frac{\pi {u_E}^2}{|\vec{u}|(|\vec{u}|-iu_4)}\ln(1-e^{-2\pi T\tau(|\vec{u}|-iu_4)})+c.c.\\
&+2\pi^2 T\tau\frac{u_4{u_E}^2}{|\vec{u}|}\left(i\int_0^1 dx\frac{1}{|\vec{u}|-iu_4 x}\frac{1}{e^{2\pi T\tau(|\vec{u}|-iu_4 x)}-1}+c.c.\right)\\
&+\pi(\lambda-1)\left[-\frac{1}{\epsilon}-\ln\frac{2\pi\mu}{\sqrt{\pi}}+\gamma+(-\gamma+\ln 4\pi T)-2\ln(1-e^{-2\pi T|\vec{u}|\tau})\right.\\
&-2|\vec{u}|\tau\pi T\frac{1}{e^{2\pi T|\vec{u}|\tau}-1}+\frac{1}{2}\ln(1-e^{-2\pi T\tau(|\vec{u}|-iu_4)})+c.c.\\
&+\pi T\tau(|\vec{u}|-iu_4)\frac{1}{e^{2\pi T\tau(|\vec{u}|-iu_4)-1}}+c.c.\\
&+3iu_4\pi T\tau\int_0^1 dx\frac{1}{e^{2\pi T\tau(|\vec{u}|-iu_4)}-1}+c.c.\\
&\left.-i2\tau^2\pi^2 T^2 u_4\int_0^1 dx(|\vec{u}|-iu_4 x)\frac{e^{-2\pi T\tau(|\vec{u}|-iu_4 x)}}{(e^{-2\pi T\tau(|\vec{u}|-iu_4 x)}-1)^2}+c.c.\right],
\end{aligned}
\tag{2.34}
$$

where $\gamma \sim 0.57\cdots$ and $E_i(-|\vec{u}|m_{sc}\tau)$ are, respectively, the Euler constant and the exponential integral function and $u_E=\sqrt{{u_4}^2+|\vec{u}|^2}$. The details are given in Appendix A.

An analytic continuation of $\mathcal{G}_E$ in Eq. (2.28) with Eq. (2.34) back to Minkowski space, which is obtained through $p_4 \longrightarrow i(p_0+i\epsilon),\ u_4 \longrightarrow i(u_0+i\epsilon)$, yields the retarded function. We normalize it through MS scheme [3], which amounts to choose the renormalization factor Z_2 as

$$
Z_2 = e^{\alpha\frac{3-\lambda}{2\pi}\frac{1}{\epsilon}}\ . \tag{2.35}
$$

Then we obtain,

$$
\mathcal{G}_R = i\int_0^\infty d\tau \exp\left\{-i\tau(M-u\cdot p-i\epsilon_c)+\frac{\alpha}{2\pi^2}\mathcal{D}_R\right\}\ , \tag{2.36}
$$

where $u\cdot p=u^\mu p_\mu$ is the scalar product in Minkowski space.

III. PHYSICAL CONTENTS OF THE TWO POINT FUNCTION

We study the behaviors of $\mathcal{D}_R$ in Eq. (2.36) in various limits of parameters involved. We divide the region $0\le\tau<\infty$ into several regions, which are discriminated by the two parameters $\tau_A=\frac{1}{|\vec{u}|T}$, $\tau_B=\frac{1}{m_{sc}|\vec{u}|}$ (see Appendix A).

a) $\tau \ll \tau_A$

$$
\mathcal{D}_R \simeq 2\pi\left\{1+\frac{3-\lambda}{2}\left(\frac{\gamma}{2}+\ln\frac{\mu\tau}{\sqrt{\pi}}-\frac{i\pi}{2}\right)\right\} \tag{3.1}
$$

b) $\tau_A \ll \tau \ll \tau_B$

$$\mathcal{D}_R \simeq 2\pi^2 T\tau \left\{ \frac{1}{|\vec{u}|} \left(\ln \tau T - \frac{i\pi}{2} + \gamma - 1 \right) - \frac{|\vec{u}|}{2}(\lambda - 1) \right\}$$
$$+2\pi \frac{3-\lambda}{2} \left(\frac{\gamma}{2} - \ln \frac{2\sqrt{\pi}T}{\mu} \right) - \pi \frac{u_0}{|\vec{u}|} \ln \frac{u_0 - |\vec{u}|}{u_0 + |\vec{u}|} - i\pi^2 \frac{u_0}{|\vec{u}|} \tag{3.2}$$

c) $\tau_B \ll \tau$

$$\mathcal{D}_R \simeq -2\pi^2 T\tau|\vec{u}| \left\{ \ln \tau T - \frac{i\pi}{2} + \gamma - 1 - \frac{{u_0}^2}{|\vec{u}|^2} \left(\ln \frac{T}{|\vec{u}| m_{sc}} - \frac{i\pi}{2} \right) + \frac{\lambda - 1}{2} \right\}$$
$$-2\pi^2 T \frac{{u_0}^2}{|\vec{u}|^2} \frac{1}{m_{sc}} + 2\pi \frac{3-\lambda}{2} \left(\frac{\gamma}{2} - \ln \frac{2\sqrt{\pi}T}{\mu} \right) - \pi \frac{u_0}{|\vec{u}|} \ln \frac{u_0 - |\vec{u}|}{u_0 + |\vec{u}|} - i\pi^2 \frac{u_0}{|\vec{u}|} \quad . \tag{3.3}$$

In the above equations, use has been made of $u^2 = 1$.

When the muon is far from the mass shell $|M - p \cdot u| \gg M$, in Eq. (2.36), the region $\tau \ll \tau_A$ gives the dominant contribution:

$$\mathcal{G}_R \simeq \frac{A\Gamma(1+\xi)}{M - u \cdot p} \left[\frac{\mu}{i(M - u \cdot p)} \right]^{\xi} , \tag{3.4}$$
$$\xi = \alpha \frac{3-\lambda}{2\pi} , \tag{3.5}$$
$$A = \exp \frac{\alpha}{\pi} \left\{ 1 + \frac{3-\lambda}{4} (\gamma - \ln \pi - i\pi) \right\} , \tag{3.6}$$

where Γ is the gamma function. When $\lambda > 3$, $\xi < 0$ in Eq. (3.4). This is just as in vacuum theory [10], which we do not discuss any further. When $\lambda = 3$, $\mathcal{G}_E$ in Eq. (3.4) behaves as the bare propagator.

We are now in a position to discuss the decaying behavior of the heavy muon. For this purpose, we should study $\mathcal{G}_R$ near on the muon mass shell $|M - p \cdot u| \sim 0$. The time dependence of the heavy-muon two-point function is obtained through the Fourier transformation;

$$G_R = \int \frac{dp_0}{2\pi} \, \mathcal{G}_R \, \exp[-ip_0 t]$$
$$\simeq \frac{M}{E} \exp[-iEt] \exp\left[\frac{\alpha}{2\pi^2} \mathcal{D}_R(\tau = Mt/E, \, u) \right] . \tag{3.7}$$

In obtaining Eq. (3.7), use has been made of the on-shell approximation $(M + \vec{u} \cdot \vec{p})/u_0 \simeq E$. From Eq. (3.7) and Eqs. (3.1)–(3.3), it is straightforward to obtain the time dependence of G_R:

a) $\tau \ll \tau_A$

$$G_R \simeq \frac{M}{E} e^{-iEt} \left(\frac{\mu M t}{\sqrt{\pi} E} \right)^{\xi} \exp\left[\frac{\alpha}{\pi} \left\{ 1 + \frac{3-\lambda}{4} (\gamma - i\pi) \right\} \right] \tag{3.8}$$

b) $\tau_A \ll \tau \ll \tau_B$

$$
\begin{aligned}
G_R \simeq \frac{M}{E} e^{-iEt} \left(\frac{MTt}{E}\right)^{\alpha \frac{MT}{E|\vec{u}|} t} \exp\Bigg[\alpha \frac{MTt}{E} \left\{\frac{1}{|\vec{u}|}\left(-\frac{i\pi}{2} + \gamma - 1\right) - \frac{|\vec{u}|}{2}(\lambda - 1)\right\} \\
+ \frac{\alpha}{\pi}\left\{\frac{3-\lambda}{2}\left(\frac{\gamma}{2} - \ln\frac{2\sqrt{\pi}T}{\mu}\right) - \frac{u_0}{2|\vec{u}|}\ln\frac{u_0 - |\vec{u}|}{u_0 + |\vec{u}|} - i\pi\frac{u_0}{2|\vec{u}|}\right\}\Bigg]
\end{aligned} \tag{3.9}
$$

c) $\tau_B \ll \tau$

$$
G_R \simeq \frac{M}{E} e^{-iEt + F(t)} \ , \tag{3.10}
$$

$$
\begin{aligned}
F(t) \equiv -\alpha \frac{MTt}{E}|\vec{u}| \left\{\ln\frac{MTt}{E} - \frac{i\pi}{2} + \gamma - 1 - \frac{{u_0}^2}{|\vec{u}|^2}\left(\ln\frac{T}{|\vec{u}| m_{sc}} - \frac{i\pi}{2}\right) + \frac{\lambda - 1}{2}\right\} \\
- \frac{\alpha}{\pi}\left\{\pi T \frac{{u_0}^2}{|\vec{u}|^2}\frac{1}{m_{sc}} - \frac{3-\lambda}{2}\left(\frac{\gamma}{2} - \ln\frac{2\sqrt{\pi}T}{\mu}\right) + \frac{u_0}{2|\vec{u}|}\ln\frac{u_0 - |\vec{u}|}{u_0 + |\vec{u}|} + i\pi\frac{u_0}{2|\vec{u}|}\right\} \ .
\end{aligned} \tag{3.11}
$$

We discuss two cases separately. It is to be noted that $|\vec{u}| \equiv \frac{1}{M}|\vec{p}| = \frac{|\vec{v}|}{\sqrt{1-|\vec{v}|^2}}$ with $\vec{v}$ being the external muon velocity:

- $0 < v < 1$
  In the region a) above, the behavior of G_R is gauge-parameter dependent, and we cannot draw any physically sensible conclusion. In the region b), the factor $\left(\frac{MTt}{E}\right)^{\alpha \frac{MT}{E|\vec{u}|} t}$ in G_R, Eq. (3.9), dominates over others. Then, the leading part of G_R is gauge-parameter independent and increases as t increases. In the region c) the dominant part of G_R is gauge-parameter independent, which first increases and then decreases. The turning point in t is approximately given by $\frac{E}{MT}(\frac{T}{m_{sc}|\vec{u}|})^{1/v^2}$.

- $v \simeq 1$
  In the region a) and b) the situation is the same as the first sentence in the above case. In the region c) G_R decreases monotonously.

Let us discuss the large t region c) more elaborately. The leading contribution comes from logarithmic parts of $F(t)$ in Eq. (3.11) (cf. Eq. (3.3)). It is to be noted that the gauge-parameter dependent part in Eq. (3.11) may be neglected and Eq. (3.11) is gauge independent:

$$
F(t) \simeq -\alpha\frac{MTt}{E}|\vec{u}|\left(\ln\frac{MTt}{E} - \frac{{u_0}^2}{|\vec{u}|^2}\ln\frac{T}{m_{sc}|\vec{u}|}\right) \ . \tag{3.12}
$$

A striking feature is that Eq. (3.12) is divergence free or finite. The damping rate γ_d is usually defined through the behavior of G_R at large t; $G_R \sim e^{-iEt - \gamma_d t}$ $(t \longrightarrow \infty)$. Ware it not for the factor $\ln t$ in Eq. (3.12), the damping rate would be identified from Eq. (3.10) with Eq. (3.12). Eq. (3.12) indicates that, at large t, the heavy muon in a QED plasma decays much faster than the *constant* damping rate. In any case, the quantity, $F(t)$, in Eq. (3.12) governs the damping behavior of the heavy muon. The behavior of $F(t)$ in Eq. (3.12) is as follows:

- $0 < v < 1$
  $F(t)$ decreases after $t = t_d \, (\equiv \frac{E}{MT}(\frac{T\sqrt{1-v^2}}{m_{sc}v})^{1/v^2})$. This means that after t_d, the heavy muon begins to decay.

- $v \simeq 1$
  In this case,

$$F(t) \simeq -\alpha \frac{MTt}{E\sqrt{1-v^2}} \ln \frac{Mm_{sc}t}{E\sqrt{1-v^2}} \, . \tag{3.13}$$

  This is always negative and the heavy muon decays.

IV. SUMMARY AND DISCUSSION

In this paper we have derived and discussed the two point function of a heavy muon passing through a QED plasma within the Bloch Nordsieck approximation. The assumption of heavy muon means that the muon line is hard. The electron sector only comes in as thermal radiative corrections to photon lines. Among those, we only take into account the HTL corrections to soft photon lines. HTL corrections to vertices are not necessary.

It is well known that, to leading order of HTL resummation scheme, the damping rate of a quark (muon) diverges due to the Coulomb-type infrared singularity even to the lowest non-trivial order of resummed perturbation theory. It is because only the electric sector of soft gluon (photon) line is screened by the thermal mass. As far as QCD is concerned, the infrared singularity is screened by the magnetic mass which is expected to arise through still higher order resummations. But in QED, no magnetic mass is induced.

In this paper, within the Bloch Nordsieck approximation, it is shown that the "damping rate" of a heavy muon in a hot QED plasma is finite. More precisely, the heavy muon decays as $t^{-\alpha \frac{MT}{E}|\vec{u}|t}$ at large t.

Finally it is worth mentioning the case where the HTL corrections to the soft photon line are ignored. (Explicit computation is given in Appendix B). For $|M - p \cdot u| \gg M$, the gauge parameter dependent parts of $\mathcal{G}_R$ (cf. Eq. (2.36)) is the same as Eq. (3.5). Also for $\lambda = 3$, $\mathcal{G}_R$ reduces essentially to the bare propagator. In the nearly on the mass shell case the time dependence of G_R is quite different from that of Eq. (3.10) with Eq. (3.12):

$$G_R \simeq \frac{M}{E} e^{-iEt} \exp \left(\frac{MT}{E} t \right)^{\frac{\alpha MTt}{E|\vec{u}|}} \quad , \tag{4.1}$$

the anti-damping behavior! From this fact, we learn that the HTL resummation for soft photon lines plays an crucial role for obtaining a physically sensible damping behavior.

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APPENDIX A: CALCULATION OF $D(\tau, U_E)$

According to the HTL resummation scheme, when the external photon lines are soft $l_4, |\vec{l}| \sim O(eT)$, the HTL corrections are necessary. Since l_4 is $2\pi nT$ $(n = 0, \pm 1, \pm 2, \cdots)$, l_4 with $n \neq 0$ is no longer in the soft region. Then we should take care of the $n = 0$ mode only.

The explicit expression for $\Pi_{T,L}$ in Eq. (2.27), within the HTL approximation, are given in [7,15]. From these expression, we have

$$\Pi_T(n=0) = 0, \;\; \Pi_L(n=0) = {m_{sc}}^2 = \frac{e^2T^2}{3} \;\; . \tag{A1}$$

We now decompose $\mathcal{D}(\tau, u_E)$ in Eq. (2.29) into two parts:

$$\begin{aligned}\mathcal{D}(\tau, u_E) &\equiv -T(2\pi\mu)^\epsilon \sum_{l_4=even} \int d^Dl \; \mathcal{D}(\tau, u_E; l_4, \vec{l}) \\ &\equiv -T(2\pi\mu)^\epsilon \int d^Dl \; \mathcal{D}(\tau, u_E; l_4, \vec{l}) \\ &\quad -T(2\pi\mu)^\epsilon \sum_{l_4=even}{}' \int d^Dl \; \mathcal{D}(\tau, u_E; l_4, \vec{l}) \; ,\end{aligned} \tag{A2}$$

where the prime on the summation symbol in Eq. (A2) means to take summation over l_4 with non-zero modes. We further decompose Eq. (A2) as

$$-T(2\pi\mu)^\epsilon \int d^Dl \; \mathcal{D}(\tau, u_E; l_4, \vec{l}) \equiv \sum_{m=1}^{3} \mathcal{D}_m \; , \tag{A3}$$

$$-T(2\pi\mu)^\epsilon \sum_{l_4=even}{}' \int d^Dl \; \mathcal{D}(\tau, u_E; l_4, \vec{l}) \equiv \sum_{m=1}^{2} \mathcal{D'}_m \; , \tag{A4}$$

where

$$\mathcal{D}_1 \equiv -T(2\pi\mu)^\epsilon \int d^Dl \; \frac{1-\cos(\vec{l}\cdot\vec{u}\tau)}{(\vec{l}\cdot\vec{u})^2} \; \frac{(u_4)^2}{\vec{l}^2 + {m_{sc}}^2} \;\; , \tag{A5}$$

$$\mathcal{D}_2 \equiv -T(2\pi\mu)^\epsilon \int d^Dl \; \frac{1-\cos(\vec{l}\cdot\vec{u}\tau)}{(\vec{l}\cdot\vec{u})^2} \; \frac{\vec{u}^2}{\vec{l}^2} \;\; , \tag{A6}$$

$$\mathcal{D}_3 \equiv -T(\lambda-1)(2\pi\mu)^\epsilon \int d^Dl \; \frac{1-\cos(\vec{l}\cdot\vec{u}\tau)}{(\vec{l}\cdot\vec{u})^2} \left(\frac{\vec{l}\cdot\vec{u}}{\vec{l}^2}\right)^2 \;\; , \tag{A7}$$

$$\mathcal{D'}_1 \equiv -T(2\pi\mu)^\epsilon \sum_{l_4=even}{}' \int d^Dl \; \frac{1-\cos(l\cdot u_E\tau)}{(l\cdot u_E)^2} \; \frac{{u_E}^2}{l^2} \;\; , \tag{A8}$$

$$\begin{aligned}\mathcal{D'}_2 \equiv -T(\lambda-1)(2\pi\mu)^\epsilon &\sum_{l_4=even}{}' \int d^Dl \; \frac{1-\cos(l\cdot u_E\tau)}{(l\cdot u_E)^2} \\ &\times \left(\frac{l\cdot u_E}{l^2}\right)^2 \; .\end{aligned} \tag{A9}$$

It is to be noted that the integrand of $\mathcal{D}_m,\ (m=1,2,3)$ and $\mathcal{D}'_m,\ (m=1,2)$ are regular at $\vec{l}=\vec{0}$.

We first evaluate $\mathcal{D}_m,\ (m=1,2,3)$. Carried out the integration over the polar angle $\cos^{-1}(\frac{\vec{l}\cdot\vec{u}}{|\vec{l}||\vec{u}|})$, Eq. (A5) becomes

$$\mathcal{D}_1 = -T(2\pi\mu)^\epsilon \frac{{u_4}^2}{|\vec{u}|^2} 2\pi^{1-\epsilon/2}\Gamma(\epsilon/2)\int_0^\infty dl\ (l^2+m_{sc}^2)^{-\epsilon}\left\{\frac{1-e^{i|\vec{u}|\tau l}}{l^2}+c.c.\right\}\ . \tag{A10}$$

In order to perform the integration over l, it is convenient to analytically continue the integrand into a complex l plane. By deforming the integration contour (as shown in Fig. 1), we obtain,

$$\begin{aligned}
\mathcal{D}_1 &= -T(2\pi\mu)^\epsilon \frac{{u_4}^2}{|\vec{u}|^2} 2\pi^{1-\epsilon/2}\Gamma(\epsilon/2)\Bigg[\frac{\pi\tau|\vec{u}|}{2}m_{sc}^{-\epsilon} - \frac{i}{2}\int_{m_{sc}}^\infty dl\frac{(l^2-{m_{sc}}^2)^{-\epsilon/2}}{l^2}e^{-i\frac{\pi}{2}\epsilon}(1-e^{-|\vec{u}|\tau l})+c.c. \\
&\quad -i\lim_{\rho\longrightarrow 0}\int_\rho^{m_{sc}} dl\frac{(l^2-{m_{sc}}^2)^{-\epsilon/2}}{l^2}(1-e^{-|\vec{u}|\tau l})+c.c.\Bigg] \\
&= -2\pi^2 T\tau\frac{{u_4}^2}{|\vec{u}|}\left(\frac{1}{\epsilon}+\ln\frac{2\pi\mu}{\sqrt{\pi}m_{sc}}-\frac{\gamma}{2}-\frac{1-e^{-|\vec{u}|\tau m_{sc}}}{|\vec{u}|\tau m_{sc}}+E_i(-|\vec{u}|\tau m_{sc})\right) \\
&= -2\pi^2 T\tau\frac{{u_4}^2}{|\vec{u}|}\left(\frac{1}{\epsilon}+\ln\frac{2\pi\mu}{\sqrt{\pi}m_{sc}}-\frac{\gamma}{2}\right) \\
&\quad +2\pi^2 T\frac{{u_4}^2}{\vec{u}^2}\begin{cases}|\vec{u}|\tau(1-\gamma-\ln|\vec{u}|\tau m_{sc})+O(|\vec{u}|^2\tau^2 m_{sc}) & (|\vec{u}|\tau m_{sc}\ll 1) \\ \dfrac{1}{m_{sc}}+O(\frac{e^{-|\vec{u}|\tau m_{sc}}}{m_{sc}}) & (|\vec{u}|\tau m_{sc}\gg 1)\end{cases}\ .
\end{aligned}\tag{A11}$$

Through similar procedures, $\mathcal{D}_2$ and $\mathcal{D}_3$ may be evaluated:

$$\mathcal{D}_2 = -2\pi^2 T|\vec{u}|\tau\left(\frac{1}{\epsilon}+\ln\frac{2\pi\mu|\vec{u}|\tau}{\sqrt{\pi}}+\frac{\gamma}{2}-1\right) \tag{A12}$$

$$\mathcal{D}_3 = -(\lambda-1)\pi^2 T|\vec{u}|\tau\ . \tag{A13}$$

The terms proportional to $1/\epsilon$, which correspond to UV divergent piece at $\epsilon=0$ or $D=3$, is to be eliminated later by renormalization.

Let us evaluate $\mathcal{D}'_m$ $(m=1,2)$ in Eq. (A4). To evaluate both Eq. (A8) and (A9), it is convenient to introduce

$$F(l_4)\equiv -T(2\pi\mu)^\epsilon\int d^D l\ \frac{1-\cos(l\cdot u_E\tau)}{(l\cdot u_E)^2}\ \frac{1}{l^2+\sigma}\ . \tag{A14}$$

The integral in Eq. (A8) is obtained from $F(l_4)$ with $\sigma=0$, while the integral in Eq. (A9) is obtained from $\partial F(l_4)/\partial\sigma$ with with $\sigma=0$. Carrying out the integrations over the polar angle and using the technique of deforming the integration contour in the l-plane, we obtain

$$F(l_4) = -T(2\pi\mu)^\epsilon\pi^{1-\epsilon/2}\Gamma(\epsilon/2)\Bigg[\frac{\pi\tau}{|\vec{u}|}\left\{l_4^2\left(1+\frac{{u_4}^2}{|\vec{u}|^2}\right)+\sigma\right\}^{-\epsilon/2}$$

$$
\begin{aligned}
&+\sin\frac{\pi\epsilon}{2}\int_{\sqrt{l_4^2+\sigma}}^{\infty} dl\ \left\{l^2-(l_4^2+\sigma)\right\}^{-\epsilon/2} \\
&\times\left\{\frac{1}{(u_4l_4+i|\vec{u}|l)^2}+c.c-\frac{e^{iu_4l_4\tau-|\vec{u}|l\tau}}{(u_4l_4+i|\vec{u}|l)^2}+c.c\right\}\Bigg] .
\end{aligned}
\tag{A15}
$$

As to the integration of the third term in last brackets in Eq. (A15), we first carry out the integration by parts and the integration by deforming the contour C_1 to $C_2 \bigoplus C_3$ in Fig. 2. Then we shift the integration valiable to get

$$
\begin{aligned}
F(l_4) = &-T(2\pi\mu)^{\epsilon}\pi^{1-\epsilon/2}\Gamma(\epsilon/2)\Bigg[\frac{\pi\tau}{|\vec{u}|}\left\{l_4^2\left(1+\frac{{u_4}^2}{|\vec{u}|^2}\right)+\sigma\right\}^{-\epsilon/2} \\
&+\sin\frac{\pi\epsilon}{2}(l_4^2+\sigma)^{-\frac{1+\epsilon}{2}}\int_0^{\infty} dl\ \frac{(l^2-1)^{-\epsilon/2}}{\left(il|\vec{u}|+\frac{lu_4}{\sqrt{l_4^2+\sigma}}\right)^2}+c.c \\
&+\frac{\tau}{|\vec{u}|}\int_{\sqrt{l_4^2+\sigma}}^{\infty} dl\ \frac{e^{-l|\vec{u}|\tau}}{l}+\frac{1}{|\vec{u}|}\frac{e^{il_4u_4\tau-|\vec{u}|\tau\sqrt{l_4^2+\sigma}}}{il_4u_4-|\vec{u}|\sqrt{l_4^2+\sigma}}+c.c. \\
&+\frac{i\tau}{|\vec{u}|}\int_0^{\frac{l_4u_4}{|\vec{u}|}} dl\ \frac{e^{-|\vec{u}|\tau(\sqrt{l_4^2+\sigma}-il)}}{\sqrt{l_4^2+\sigma}-il}+c.c.\Bigg] .
\end{aligned}
\tag{A16}
$$

Using Eq. (A16) with $\sigma=0$ and carrying out the summation over l_4, we obtain

$$
\begin{aligned}
\mathcal{D'}_1 = &\sum_{l_4=even}{}' {u_E}^2F(l_4)\Big|_{\sigma=0} \\
= &\ 2\pi^2T\tau\frac{{u_E}^2}{|\vec{u}|}\left(\frac{1}{\epsilon}-\frac{\gamma}{2}-\ln\sqrt{\pi}\right)+\frac{2\pi}{\epsilon}+\pi\gamma-2\pi\ln\frac{4\pi^{3/2}T}{2\pi\mu}-\frac{2\pi u_4}{|\vec{u}|}\tan^{-1}\frac{u_4}{|\vec{u}|} \\
&-2\pi^2T\tau\frac{{u_E}^2}{|\vec{u}|}\left(\ln\frac{T}{2\pi\mu}+\ln\frac{u_E}{|\vec{u}|}\right)+4\pi^2T\tau\frac{{u_E}^2}{|\vec{u}|}\int_1^{\infty}dx\ \frac{1}{x}\frac{1}{e^{2\pi T|\vec{u}|\tau x}-1} \\
&+\frac{\pi{u_E}^2}{|\vec{u}|(|\vec{u}|-iu_4)}\ln(1-e^{-2\pi T\tau(|\vec{u}|-iu_4)})+c.c. \\
&+2\pi^2T\tau\frac{u_4{u_E}^2}{|\vec{u}|}\left(i\int_0^1 dx\ \frac{1}{|\vec{u}|-iu_4x}\ \frac{1}{e^{2\pi T\tau(|\vec{u}|-iu_4x)}-1}+c.c.\right) .
\end{aligned}
\tag{A17}
$$

$\mathcal{D'}_2$ in Eq. (A9) may be evaluated as:

$$
\mathcal{D'}_2 = -(\lambda-1)\sum_{l_4=even}{}'\left[T(2\pi\mu)^{\epsilon}\int d^Dl\frac{1}{l^4}-\frac{\partial^3}{\partial\sigma\partial\tau^2}F(l_4)\Big|_{\sigma=0}\right] ,
\tag{A18}
$$

and we obtain,

$$
\mathcal{D'}_2 = -(\lambda-1)\pi\left[+\frac{1}{\epsilon}+\ln\frac{2\pi\mu}{\sqrt{\pi}}-\gamma+(\gamma-\ln 4\pi T)+2\ln(1-e^{-2\pi T|\vec{u}|\tau})+2|\vec{u}|\tau\pi T\frac{1}{e^{2\pi T|\vec{u}|\tau}-1}\right.
$$

$$
\begin{aligned}
&-\frac{1}{2}\ln(1-e^{-2\pi T\tau(|\vec{u}|-iu_4)})-c.c.\\
&-\pi T\tau(|\vec{u}|-iu_4)\frac{1}{e^{2\pi T\tau(|\vec{u}|-iu_4)-1}}-c.c.\\
&-3iu_4\pi T\tau\int_0^1 dx\frac{1}{e^{2\pi T\tau(|\vec{u}|-iu_4)}-1}+c.c.\\
&+i2\tau^2\pi^2T^2u_4\int_0^1 dx\,(|\vec{u}|-iu_4x)\frac{e^{-2\pi T\tau(|\vec{u}|-iu_4x)}}{(e^{-2\pi T\tau(|\vec{u}|-iu_4x)}-1)^2}+c.c.\Bigg]\,. \qquad \text{(A19)}
\end{aligned}
$$

Now let us study the behaviors of $\mathcal{D'}_m$ ($m=1,2$) in the following two limits (τ_A is as in the text, $\tau_A=\frac{1}{|\vec{u}|T}$):

a) $\tau \ll \tau_A$

$$
\mathcal{D'}_1 \simeq 2\pi\left(\frac{1}{\epsilon}+\frac{\gamma}{2}+1+\ln\frac{2\pi\mu u_E\tau}{2\sqrt{\pi}}\right)+2\pi^2\tau T\frac{{u_E}^2}{|\vec{u}|}\left(\frac{1}{\epsilon}+\frac{\gamma}{2}-1+\ln\frac{2\pi\mu|\vec{u}|\tau}{\sqrt{\pi}}\right)\ , \qquad \text{(A20)}
$$

$$
\mathcal{D'}_2 \simeq -(\lambda-1)\pi\left(\frac{1}{\epsilon}+\frac{\gamma}{2}+1+\ln\frac{2\pi\mu u_E\tau}{2\sqrt{\pi}}\right)+(\lambda-1)\pi^2T|\vec{u}|\tau\ , \qquad \text{(A21)}
$$

and,

b) $\tau \gg \tau_A$

$$
\mathcal{D'}_1 \simeq 2\pi\left(\frac{1}{\epsilon}+\frac{\gamma}{2}-\ln\frac{4\pi^{3/2}T}{2\pi\mu}-\frac{u_4}{|\vec{u}|}\tan^{-1}\frac{u_4}{|\vec{u}|}\right)+2\pi^2T\tau\frac{{u_E}^2}{|\vec{u}|}\left(\frac{1}{\epsilon}-\frac{\gamma}{2}+\ln\frac{2\pi\mu|\vec{u}|}{u_E\sqrt{\pi}T}\right), \qquad \text{(A22)}
$$

$$
\mathcal{D'}_2 \simeq -(\lambda-1)\pi\left(\frac{1}{\epsilon}+\frac{\gamma}{2}-\ln\frac{4\pi^{3/2}T}{2\pi\mu}\right)\ . \qquad \text{(A23)}
$$

Substituting all the results obtained above into Eq. (A2), we deduce the following behaviors of $\mathcal{D}(\tau,u_E)$ in various limits (τ_B is as in the text, $\tau_B=\frac{1}{m_{sc}|\vec{u}|}$):

a) $\tau \ll \tau_A$

$$
\mathcal{D}(\tau,u_E) \simeq 2\pi\left(\frac{1}{\epsilon}+\frac{\gamma}{2}+1+\ln\frac{2\pi\mu\tau u_E}{2\pi\sqrt{\pi}}\right)-\pi(\lambda-1)\left(\frac{1}{\epsilon}+\frac{\gamma}{2}+\ln\frac{2\pi\mu\tau u_E}{2\pi\sqrt{\pi}}\right)\ , \qquad \text{(A24)}
$$

b) $\tau_A \ll \tau \ll \tau_B$

$$
\begin{aligned}
\mathcal{D}(\tau,u_E) \simeq\ & -2\pi^2T\tau\frac{{u_E}^2}{|\vec{u}|}(\ln\tau Tu_E+\gamma-1)+2\pi\left(\frac{1}{\epsilon}+\frac{\gamma}{2}-\ln\frac{4\pi^{3/2}T}{2\pi\mu}-\frac{u_4}{|\vec{u}|}\tan^{-1}\frac{u_4}{|\vec{u}|}\right)\\
& -(\lambda-1)\pi^2T|\vec{u}|\tau-(\lambda-1)\pi\left(\frac{1}{\epsilon}+\frac{\gamma}{2}-\ln\frac{4\pi^{3/2}T}{2\pi\mu}\right)\ , \qquad \text{(A25)}
\end{aligned}
$$

c) $\tau_B \ll \tau$

$$\mathcal{D}(\tau,u_E) \simeq -2\pi^2 T\tau\frac{{u_E}^2}{|\vec{u}|}(\ln\tau T u_E+\gamma-1)+2\pi\left(\frac{1}{\epsilon}+\frac{\gamma}{2}-\ln\frac{4\pi^{3/2}T}{2\pi\mu}-\frac{u_4}{|\vec{u}|}\tan^{-1}\frac{u_4}{|\vec{u}|}\right)$$
$$-(\lambda-1)\pi^2 T|\vec{u}|\tau-(\lambda-1)\pi\left(\frac{1}{\epsilon}+\frac{\gamma}{2}-\ln\frac{4\pi^{3/2}T}{2\pi\mu}\right)$$
$$-2\pi^2 T|\vec{u}|\tau\frac{{u_4}^2}{|\vec{u}|^2}(-\gamma+1-\ln|\vec{u}|\tau m_{sc})+2\pi^2 T\frac{{u_4}^2}{|\vec{u}|^2}\frac{1}{m_{sc}} \quad . \tag{A26}$$

The remaining tasks are to eliminate UV divergence through renormalization with MS scheme and to perform the τ integration (see text).

APPENDIX B: NO HTL CORRECTION TO THE PHOTON PROPAGATOR

We set $\Pi_{T,L}=0$ in Eq. (2.29). In this case, Eq. (2.29) reduces to

$$\mathcal{D}(\tau,u_E)\Big|_{m_{sc}=0} \equiv \mathcal{D}_{1+2}\Big|_{m_{sc}=0}+\mathcal{D}_3\Big|_{m_{sc}=0}+\sum_{m=1}^{2}\mathcal{D}'_m\Big|_{m_{sc}=0} \tag{B1}$$
$$= -T(2\pi\mu)^\epsilon\int d^D l\frac{1-\cos(\vec{l}\cdot\vec{u}\tau)}{(\vec{l}\cdot\vec{u})^2}$$
$$\times\left\{\frac{{u_E}^2}{\vec{l}^2}+(\lambda-1)\left(\frac{\vec{l}\cdot\vec{u}}{\vec{l}^2}\right)^2\right\} \quad . \tag{B2}$$

Here

$$\mathcal{D}_{1+2}\Big|_{m_{sc}=0} \equiv -T(2\pi\mu)^\epsilon\int d^D l\;\frac{1-\cos(\vec{l}\cdot\vec{u}\tau)}{(\vec{l}\cdot\vec{u})^2}\frac{{u_E}^2}{\vec{l}^2} \quad , \tag{B3}$$
$$\mathcal{D}_3\Big|_{m_{sc}=0} \equiv -(\lambda-1)T(2\pi\mu)^\epsilon\int d^D l\;\frac{1-\cos(\vec{l}\cdot\vec{u}\tau)}{(\vec{l}\cdot\vec{u})^2}\left(\frac{\vec{l}\cdot\vec{u}}{\vec{l}^2}\right)^2 \quad . \tag{B4}$$

Similar procedures as in Appendix A leads to

$$\mathcal{D}_{1+2}\Big|_{m_{sc}=0} \simeq -2\pi^2\tau T\frac{{u_E}^2}{|\vec{u}|}\left(\frac{1}{\epsilon}+\frac{\gamma}{2}-1+\ln\frac{2\pi\mu|\vec{u}|\tau}{\sqrt{\pi}}\right) \quad , \tag{B5}$$
$$\mathcal{D}_3\Big|_{m_{sc}=0} \simeq -(\lambda-1)\pi^2 T|\vec{u}|\tau \; . \tag{B6}$$

Substituting the above Eqs. (B5) and (B6) and Eqs. (A20)–(A23) to Eq. (B1), we have
a) $\tau \ll \tau_A$

$$\mathcal{D}(\tau,u_E)\Big|_{m_{sc}=0} \simeq 2\pi\left(\frac{1}{\epsilon}+\frac{\gamma}{2}+1+\ln\frac{2\pi\mu u_E\tau}{2\sqrt{\pi}}\right)-\pi(\lambda-1)\left(\frac{1}{\epsilon}+\frac{\gamma}{2}+\ln\frac{2\pi\mu u_E\tau}{2\sqrt{\pi}}\right) \quad . \tag{B7}$$

b) $\tau \gg \tau_A$

$$
\begin{aligned}
\mathcal{D}(\tau, u_E)\Big|_{m_{sc}=0} \simeq & -2\pi^2 T\tau \frac{{u_E}^2}{|\vec{u}|}\left(\ln T u_E \tau + \gamma - 1\right) + 2\pi\left(\frac{1}{\epsilon} + \frac{\gamma}{2} - \ln\frac{4\pi^{3/2}T}{2\pi\mu} - \frac{u_4}{|\vec{u}|}\tan^{-1}\frac{u_4}{|\vec{u}|}\right) \\
& -\pi^2(\lambda - 1)T|\vec{u}|\tau - \pi(\lambda - 1)\left(\frac{1}{\epsilon} + \frac{\gamma}{2} - \ln\frac{2\pi\mu u_E \tau}{2\sqrt{\pi}}\right) \ .
\end{aligned} \tag{B8}
$$

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FIGURES

FIG. 1. Integration contour in the complex l plane

FIG. 2. The integration along the contour C_1 may be evaluated along the contour $C_2 \bigoplus C_3$.

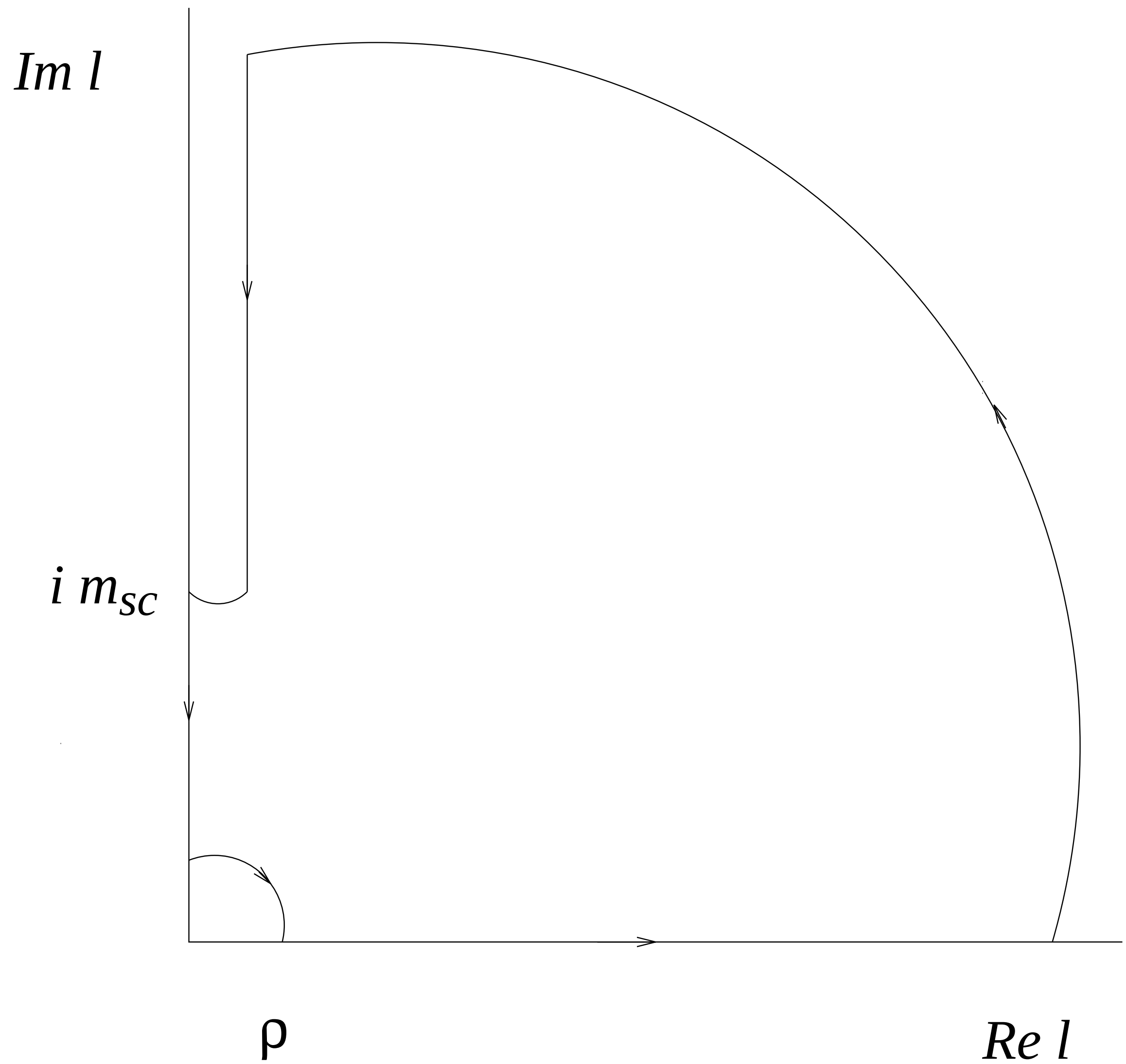

Fig. 1

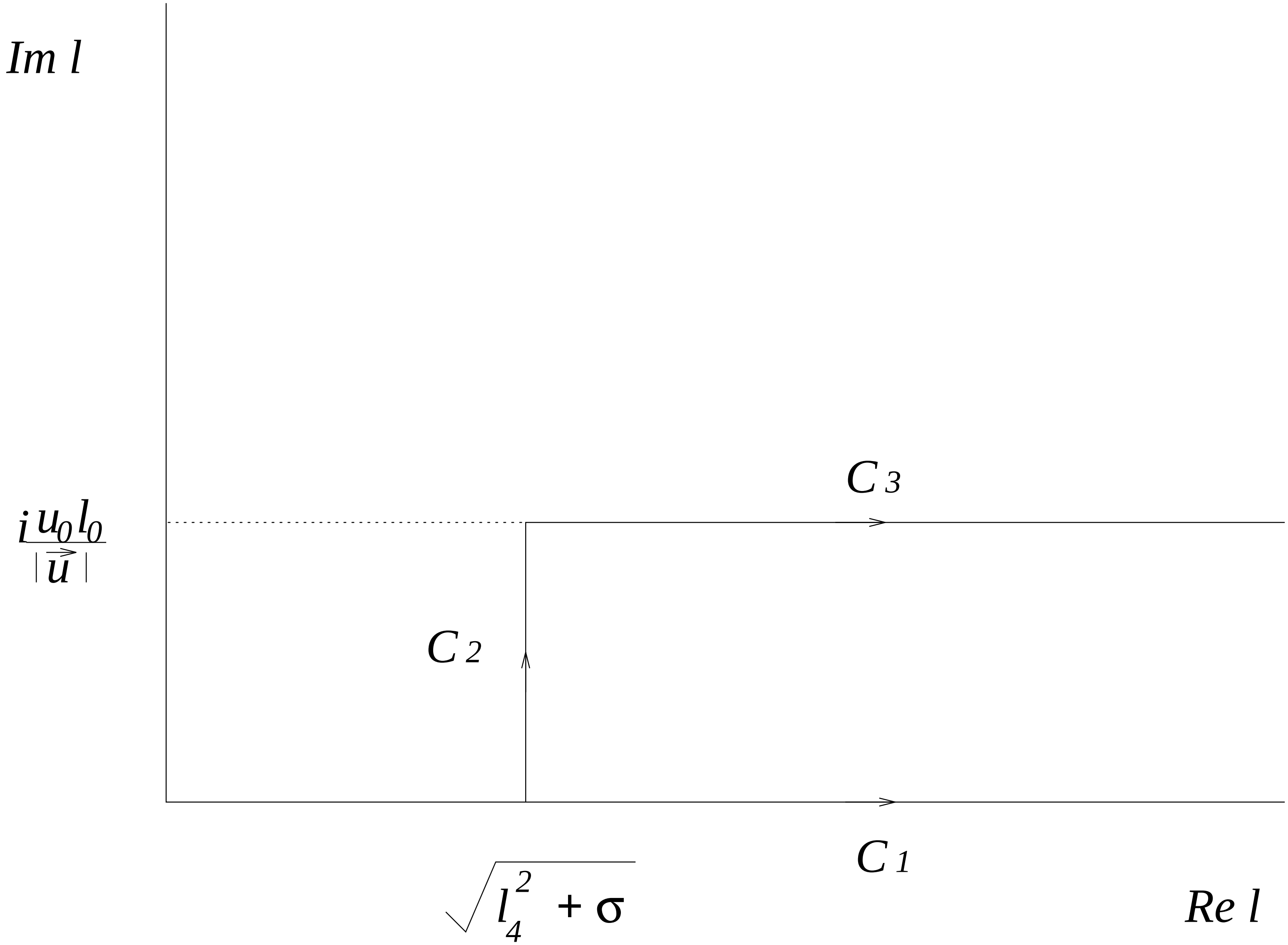

Fig. 2